The Navier-Stokes Equation, The Boltzman Transport Equation and The Convection-Diffusion Equation: An Alternative Approach to Understanding Stock Price Movements

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1. Introduction

Following Eckmann and Ruelle (1985), the motion of a viscous fluid in a bounded container Ω is given by the Navier-Stokes equation

$$\delta v_i/\delta t = -\Sigma v_j \delta_j v_i + \upsilon \Delta v_j - \delta_i p/d + g_i \qquad (1)$$

where v is the velocity field, υ is kinematic viscosity, d is density, p is pressure and g is external force field.

The Boltzmann Equation or Boltzmann Transport Equation (BTE) describes the statistical behaviour of a thermodynamic system not in a state of equilibrium. The equation arises not by analyzing the individual positions of each particle in the fluid, but by considering a probability distribution for the position and momentum of a typical particle.

So $r + \Delta r = r + p\Delta t/m$

where r is the initial position of the particle, p is pressure, m is mass and t is time. From there it is derived that

$$df/dt = (\delta f/\delta t)_{force} + (\delta f/\delta t)_{diffusion} + (\delta f/\delta t)_{collision} \qquad (2)$$

The force term represents external forces on the particles, diffusion represents diffusion of particles and collision represents forces acting between particles in collisions.

(See McGraw Hill Encyclopaedia of Physics (2nd Edition), C. B. Parker, 1994, ISBN 0-07-051400-3 and Wikipedia)

The Convection–Diffusion equation describes physical phenomena where particles, energy, or other physical quantities are transferred inside a physical system due to two processes: diffusion and convection.

The general equation is

$$\delta c/\delta t = \nabla.(D\nabla c) - \nabla.(vc) + R \qquad (3)$$

where

c is the variable of interest, D is the diffusion coefficient, v is the velocity field that the quantity is moving with and R describes creation or destruction of c. For heat transport, $R > 0$ might occur if thermal energy is being generated by friction.That is, some addition or subtraction that arises in the process. ∇ represents gradient and $\nabla\cdot$ represents divergence.

The right-hand side of equation (3) is the sum of three components. The first, $\nabla\cdot(D\nabla c)$, describes diffusion. Imagine that c is the concentration of a chemical. When concentration is low compared to the surrounding areas (e.g. a local minimum of concentration), the substance will diffuse in from the surroundings, so the concentration will increase. Conversely, if concentration is high compared to the surroundings (e.g. a local maximum of concentration), then the substance will diffuse out and the concentration will decrease.

The second contribution, $-\nabla\cdot(vc)$, describes convection (or advection). Imagine standing on the bank of a river, measuring the water's salinity (amount of salt) each second. Upstream, somebody dumps a bucket of salt into the river. A while later, you would see the salinity suddenly rise, then fall, as the zone of salty water passes by. Thus, the concentration at a given location can change because of the flow.

(See Introduction to Climate Modelling, Advances in Geophysical and Environmental Mechanics and Mathematics, Thomas Stocker, 2011, Springer, and Wikipedia)

Movement in stock prices and their prediction has been of great interest to researchers and financial market players, and the latter survive on the basis of taking positions in the market in financial assets based on their estimation of future movement of prices of the assets. Starting from financial services companies, mutual funds, insurance companies and hedge funds right up to individual traders, each market player tries to generate positive returns from estimating directional movement in stock prices. Besides fundamental performance of companies, technical analysis of stock prices has been the basis of taking market positions by players. These are methods developed by individual traders, and have gained traction over time through their usage and revealed results.

Markov processes and Brownian motion have also been applied to understand movement in stock prices over time. Further, the literature has seen extensive application of econometric methods and machine learning tools in forecasting stock prices. The interested reader can refer to Basu (1977, 1983), Jaffe et al. (1989), Strong (1993), Banz (1981), Rosenberg et al. (1985), Chan et al. (1991), Fama and French (1988, 1992, 1995), Strong and Xu (1997), Chui and Wei (1998), Bhandari (1988), Kothari and Shanken (1993), Ibbotson and Idzorek (1998), Jarrett and Kyper (2011), Adebiyi et al. (2014), Mondal et al. (2014), Singh (2015), Srinivasan and Prakasam (2014), Mishra (2016), Mostafa (2010), Kimoto et al. (1990), Jaruszewicz and Mandziuk (2004), Refenes et al. (1994), Dutta et al. (2006), Shen et al. (2007), Pan et al. (2005), Chen et al. (2003), Wu et al. (2008), Perez-Rodriguez et al. (2005), Siddiqui and Abdullah (2015),Kostadinova et al (2021), Azizah et al (2020), Guloksuz (2021), Venugopal et al (2020), Dai et al (2014), Samuel and Ekon (2020), Agustini et al (2018), Si and Bishi (2020) and their references for further details.

This paper is an attempt in understanding stock price movements from concepts in physics explaining motion,and we use the ideas expressed in the above three equations for the purpose.

2. Stock Market Constructs

The variables that emerge from the above three equations explainingmovement are viscosity, density, pressure, external force field, force, diffusion, collision, velocity field, creation and destruction and convection. We are interested in the dynamics of stock price movements, and in Table 1 we define some constructs from the stock market that can approximate the above variables.

Table 1: The variables in the equations and their stock market counterparts

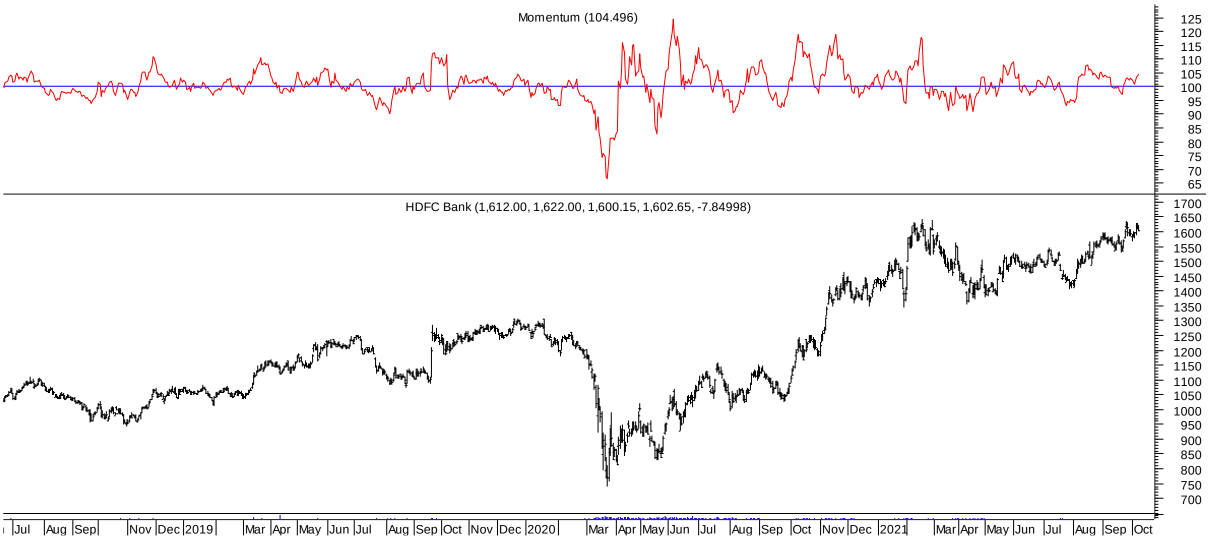
	Variables following from the equations	Stock market equivalents
1	Velocity	Momentum = (P_t – P_{t-5}), where P is stock price and the subscript is time.
2	Viscosity	a. Standard Deviation of the stock price b. Larry William's %R %R = (Highest High − Close) / (Highest High− Lowest Low) where Highest High = Highest price in the last 14 days Close = Most recent closing price Lowest Low = Lowest price in the last 14 days.
3	Density	Volume of trade /(% free float*paid up capital)
4	Pressure	Chaikin Oscillator (CO) = n-day Sum of [{((C - L) - (H - C)) / (H - L)}* Volume] / n-day Sum of Volume where n = number of periods, typically 21 H = high price in a day L = lowprice in a day C = close price in a day Volume = number of shares traded during a day
5	External Force Field/Velocity Field	a. Volatility in Nifty (the stock market sentiment index in India) b. VIX (the implied volatility index in India) c. Volatility in DJIA (the stock market sentiment index in the US) d. Volatility in crude oil prices e. Volatility in the exchange rate
6	Force	Volume of trade *ΔMomentum
7	Diffusion	a. Parabolic SAR (PSAR) Uptrend: PSAR = Prior PSAR + Prior AF (Prior EP – Prior PSAR) Downtrend: PSAR = Prior PSAR – Prior AF (Prior PSAR – Prior EP), Where: EP = Highest high for an uptrend and lowest low for a downtrend, updated each time a new EP is reached.

		AF = Default of 0.02, increasing by 0.02 each time a new EP is reached, with a maximum of 0.20 b. Diffusion Index – how many of NSE stocks are trading above their 50 DMA. DiffusionIndex (DI)=(Advances−Declines)+PDIV where Advances=Number of stocks moving higher Declines=Number of stocks moving lower PDIV=Previous DI value Generally the stocks should belong to the same sector
8	Collision	Moving Average Convergence Divergence (MACD) and the Trigger Line
9	Creation	Creation of Interest a. 15 DMA intersecting 30 DMA from below b. The Accumulation/Distribution Indicator (A/D) Formula MFM= [(Close−Low)−(High−Close)]/High−Low where MFM=Money Flow Multiplier Close=Closing price Low=Low price for the period High=High price for the period Money Flow Volume=MFM×Volume A/D=Previous A/D+CMFV where CMFV= Money flow volume c. Announcement of take-over/spin-off/merger
10	Destruction	Destruction of Interest a. 200 DMA intersecting 100 DMA from above b. Falling A/D Line c. Announcement of take-over/spin-off/merger
11	Convection	Effect of news like annual results, announcement of dividend, appointment of new CEO

3. Explanation of the Stock Market Constructs and their Visualization

In this section we try to establish the relationship between the variables in the three equations in physics with the stock market constructs that we have described in Section 2. The variable that we are interested in is time varying movement of stock prices or its velocity, and we use momentum as a proxy. This is the dependent variable. Figure 1 shows momentum for stock price of HDFC Bank over time. The lower panel is the time varying movement in the stock price, and the upper panel is momentum. Our interest is in understanding the top panel.

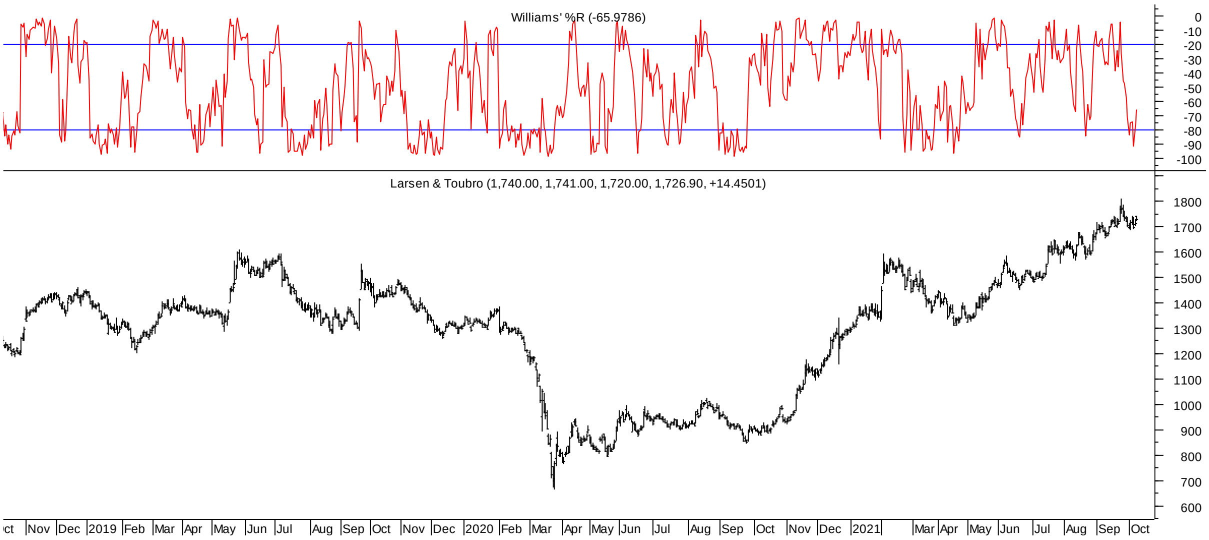
Figure 1: Movement in Momentum over time for HDFC Bank



Source: Metastock

To represent viscosity, we choose standard deviation of stock price returns and a technical indicator, William's %R. Viscosity is thickness of a fluid and it affects velocity. In stock markets, the extent of intraday variance of prices, and also variance over time, indicates investor interest in the stock leading to buying and selling. If prices are stable throughout a day without much intraday variance, it implies less interest in the stock by investors. Thus, lower the variance and also lower the range, lower would be momentum. Stocks with greater volatility in prices and greater range imply investor interest, and hence has greater momentum. Figure 2 shows William's %R in the upper panel for the stock price of Larsen and Toubro in the lower panel.

Figure 2: Movement in William's %R for Larsen and Toubro

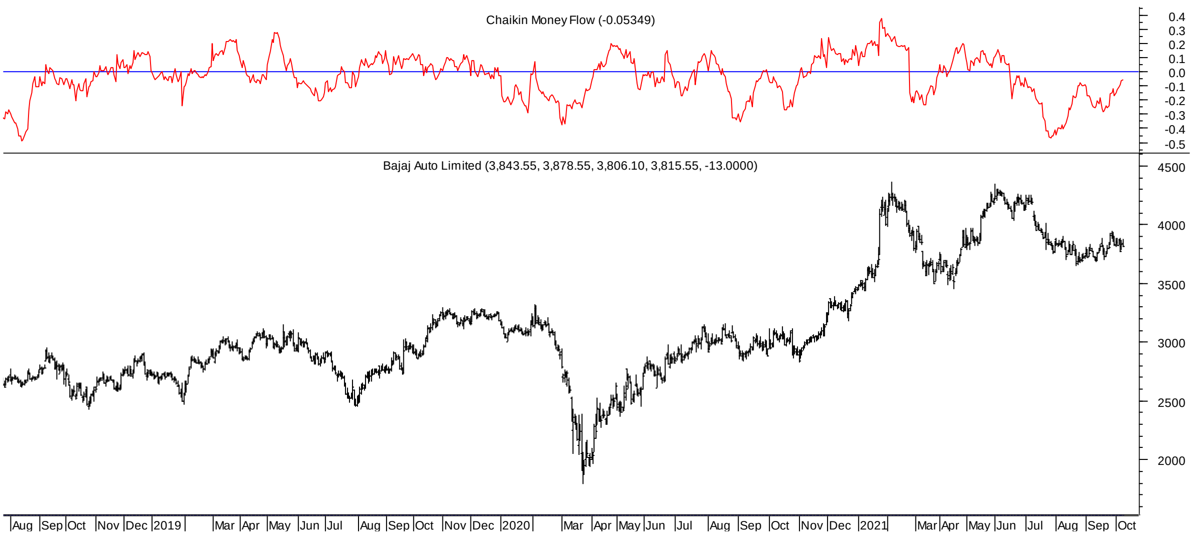


Source: Metastock

As a proxy for density, we use volume of trade / (% free float*paid up capital). It indicates total daily volume of trade as a percentage of shares that can be traded. This ratio will generally be less than one as it is quite safe to assume that all shares available for trading will not actually be traded every day. If the term in the bracket is small implying that there are lower number of shares that are available for trading, this low availability will make sensitivity of prices to changes in demand quite high.

We interpret pressure as buying and selling pressure and use Chaikin Money Flow, a technical indicator as proxy. With buying pressure money will flow into a stock and prices will rise. With selling pressure, money will flow out from a stock and prices will fall. Figure 3 shows Chaikin Money Flow in the upper panel and the stock price movement of Bajaj Auto in the lower panel.

Figure 3: Chaikin Money Flow and its movement for Bajaj Auto

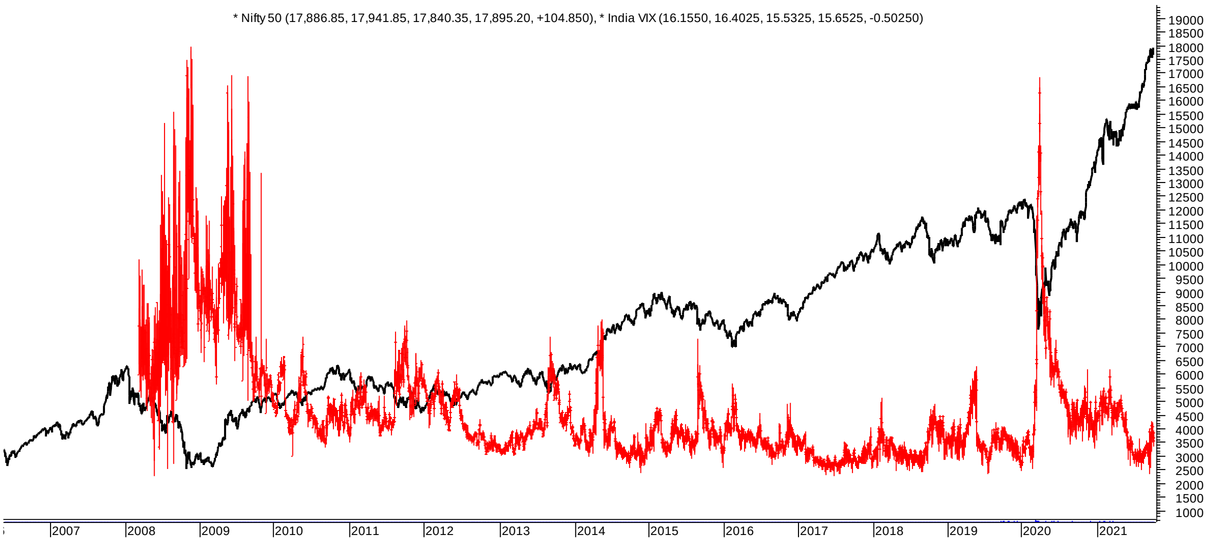


Source :Metastock

The variables listed under external force field reflect the macroeconomic environment and its change within which companies operate. These influence market perception about companies as external factors like exchange rate fluctuations or rise in crude oil prices can affect profitability of companies. Figure 4 presents one such external force, India VIX, which is the implied volatility in the stock market as derived from the options market.

Implied Volatility (IV) is the measure of volatility that is obtained from the Black and Scholes option pricing model. Volatility is the standard deviation of the returns of the underlying. If the underlying is NIFTY, then volatility means the standard deviation of NIFTY returns. The option price is a function of the spot price S, the strike price K, time to expiry t, volatility σ and the rate of interest r. For a particular option strategy if we plug in all these values, then we will get a theoretical value of the option price. At any point of time, S, K, t and r are observable. Only volatility has to be calculated. This implies that we will have to plug in a value of σ. The only value that we will have in hand is the value of Historic Volatility (HV) of the underlying. However, if we plug in the actual option price that is quoted in the market, we will get a value of σ from the options formula; this is IV. This is the volatility as implied by the options market. Since it is derived by taking the help of the actual quoted options price, it is volatility of the underlying that is expected in the future. In the stock exchanges an index of volatility is available, which is constructed after taking a weighted average of IV of different kinds of options contracts. In India we have India VIX. Whaley (2009) has provided the intuition behind VIX. According to him “It is important to emphasize that the VIX is forward looking; that is, it measures volatility that investors expect to see. It is not a of backward-looking index to measure volatility that has been recently realized…… the level of VIX is implied by the current prices on options and represents future stock market volatility…”

Figure 4: Movement in India VIX and Nifty



Source: Metastock

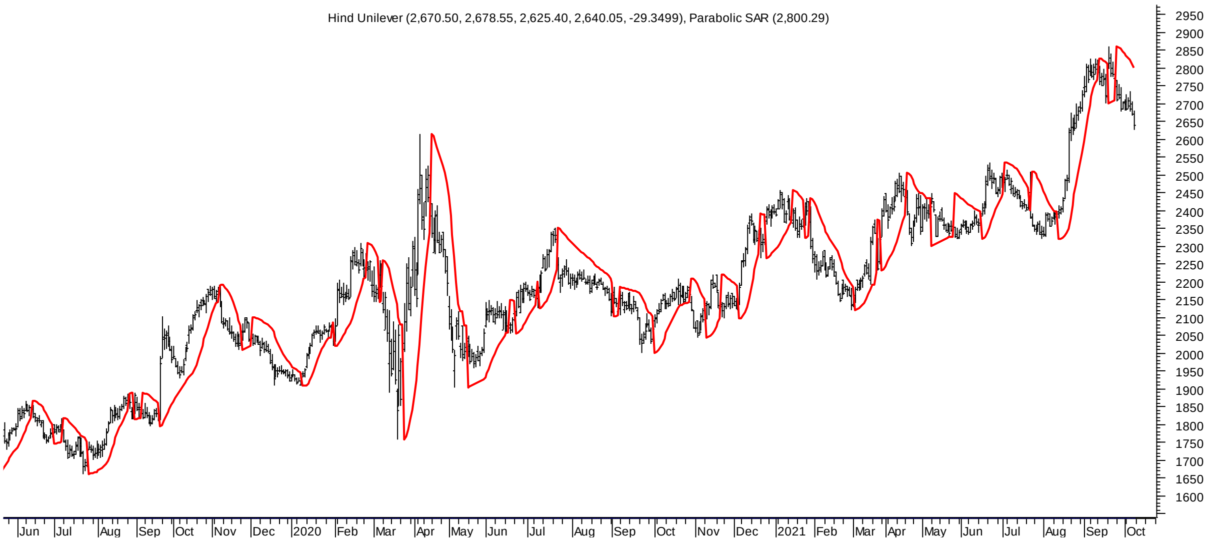
In Figure 4, the black line shows movement in Nifty and the red line shows movement in the implied volatility index, India VIX. VIX is an indicator of market fear and observe that there is an inverse relationship between India VIX and Nifty. India VIX shot up during the global financial crisis of 2008-09 and during the onset of the pandemic in the beginning of 2020. There have been intermittent spikes in VIX also due to macroeconomic shocks. This external factor we take as a proxy for external force field.

In the Convection – Diffusion equation, there is a concept of velocity field with which the particle is moving. This is an external factor and we represent this also by our external factors, in particular, through the variability of the constructs. We are thus able to model, not only effects of actual shocks in the external world, but also expected shocks.

We represent force by Volume of Trade multiplied by change in Momentum. It implies that if the rate of change of price is increasing, that is the second partial is positive, and if the volume of trade is significant, then this would lead to a significant increase in stock price in the future.

An appropriate proxy for diffusion would be Parabolic SAR. A look at Figure 5 would explain our choice of this technical indicator.

Figure 5: Parabolic SAR for Hindustan Unilever



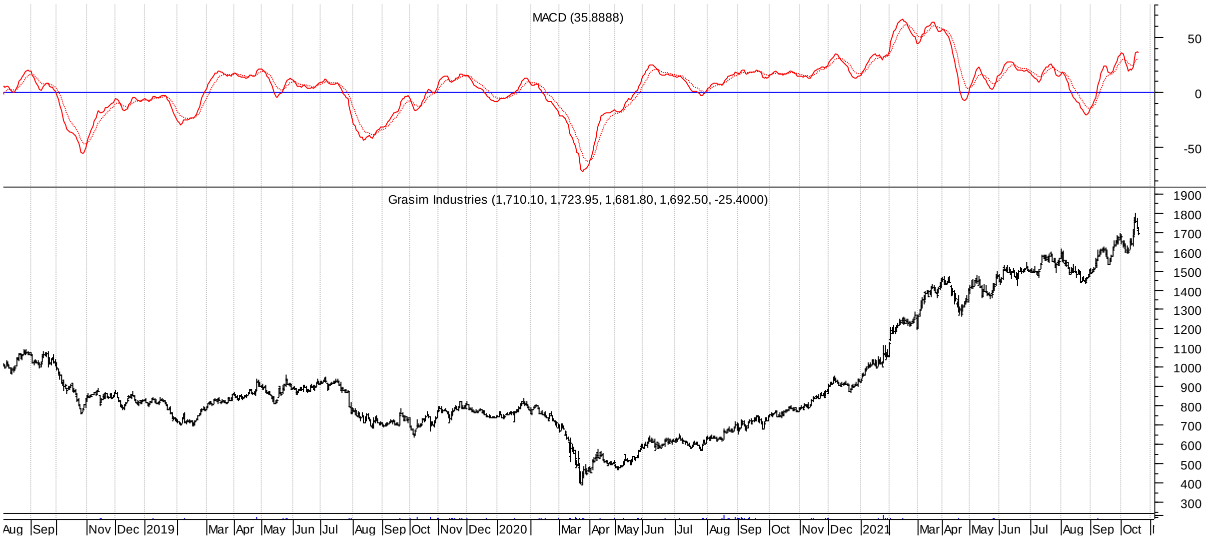
Source: Metastock

The red line is Parabolic SAR and the dark line is the stock price movement of Hindustan Unilever. When a trend continues in the dark line, the red line catches up fats with it, and then moves away from the dark line. It gives buy/sell signals, but observe how the red line always approaches the dark line, and every time moves away from it. This we consider as diffusion.

We also use a diffusion index for the market as a whole. However, this is defined for a group of stocks and is a better indicator for the performance of the sector or the market as a whole.

For collision, we use a technical indicator called Moving Average Convergence Divergence (MACD) and its intersection with the trigger line. MACD is defined as the difference between 26 Day Exponential Moving Average (EMA) and the 12 Day EMA. We then construct a 9 Day EMA of the spread. The latter is called the trigger line. When the MACD line intersects the trigger line from below it is a buy signal, and when it intersects from above it is a sell signal. Further, the MACD line above zero is an overbought position, and it being smaller than zero is an oversold position. Figure 6 presents the diagram for Grasim Industries Ltd. The dark red line in the upper panel is the MACD line and the dotted line is the trigger line. One can observe that whenever the two lines come close and intersect, there is a reversal of position. Thus, when the lines collide, there is a reversal in movement of prices.

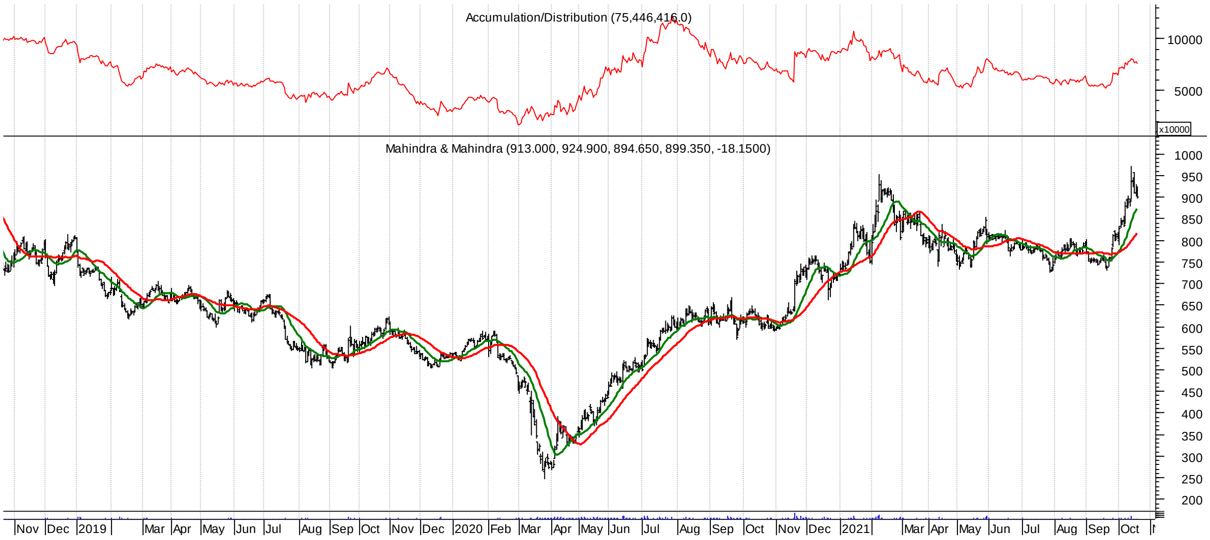
Figure 6: MACD for Grasim Industries Ltd.



Source: Metastock

We interpret the variable creation, as creation of interest in a stock, and we use 15 DMA of the stock price intersecting 30 DMA of the stock price from below, and the Accumulation/Distribution Indicator (A/D). We feel interest can created quickly, and we use intersection of the short period DMAs for the purpose. The AD line is a cumulative concept and hence is long term by nature. If interest is being generated in a stock, it must get reflected in the range of prices and also in the accompanying volume. This added with previous day’s indicator value, gives it a cumulative sustaining interpretation. Figure 7 provides examples of these indicators for Mahindra & Mahindra Ltd. The top panel shows the A/D line and the bottom panel shows the stock price in black, the 15 DMA in green and 30 DMA in red.

Figure 7: Moving Average and A/D line



Source: Metastock

We interpret the factor destruction as destruction of interest in a stock. As different from creation of interest, destruction is a fairly long phenomenon and takes time to build. We consider intersection of the 200 Day Moving Average (DMA) intersecting the 100 DMA from above, to indicate destruction. The 200 DMA is quite a long period average, and it falling and intersecting the 100DMA from above, shows gradual loss of interest of the market in the stock. Figure 8 provides an example of such a phenomenon for ONGC where the dark line is the stock price, the blue line 200 DMA and the green line is 100 DMA. For such stocks, it takes time for the stock prices to recover.

Figure 8: Moving Average for BHEL

Source: Metastock

Convection is temporary shocks to the flow of a variable, and the effect of these shocks die down after some time. We feel that effect of news like annual results, announcement of dividend, appointment of new CEO can be proxies for convection in the stock market. This cannot be represented by any technical indicator, but can be studied through discontinuity in movement of stock prices in the presence of shocks. There are studies like the Monday Effect, and the impact is temporary. This has little to do with the fundamental functioning of the company, but causes a temporary ripple.

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